SOLUTION OF EXERCISE # 6.2

Exercise # 6.2

Q.1: Find $\vec{a}.\vec{b}$ and $\vec{a}\times\vec{b}$:

(i) $\vec{a} = 2i + 3j + 4k,\quad \vec{b} = i - j + k$

Sol. (IIA-2017), (IIA-2019), (IA-2021)

$$\vec{a}.\vec{b}$$
$$=(2i+3j+4k).(i-j+k)$$
$$=(2)(1)+(3)(-1)+(4)(1)$$
$$=2-3+4$$
$$=\boxed{3}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$
$$= i\begin{vmatrix} 3 & 4 \\ -1 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$$
$$= i(3+4) - j(2-4) + k(-2-3)$$
$$= i(7) - j(-2) + k(-5) = \boxed{7i + 2j - 5k}$$

(ii) $\vec{a} = i + j + k,\quad \vec{b} = -5i + j - 3k$

Sol.

$$\vec{a}.\vec{b}$$
$$=(i+j+k).(-5i+2j-3k)$$
$$=(1)(-5)+(1)(2)+(1)(-3)$$
$$=-5+2-3$$
$$=\boxed{-6}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ -5 & 2 & -3 \end{vmatrix}$$
$$= i\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ -5 & -3 \end{vmatrix} + k\begin{vmatrix} 1 & 1 \\ -5 & 2 \end{vmatrix}$$
$$= i(-3-2) - j(-3+5) + k(2+5)$$
$$= i(-5) - j(2) + k(7) = \boxed{-5i - 2j + 7k}$$

(iii) $\vec{a} = -i - j - k,\quad \vec{b} = 2i + j$

Sol.

$$\vec{a}.\vec{b}$$
$$=(-i-j-k).(2i+j)$$
$$=(-1)(2)+(-1)(1)+(-1)(0)$$
$$=-2-1-0$$
$$=\boxed{-3}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ -1 & -1 & -1 \\ 2 & 1 & 0 \end{vmatrix}$$
$$= i\begin{vmatrix} -1 & -1 \\ 1 & 0 \end{vmatrix} - j\begin{vmatrix} -1 & -1 \\ 2 & 0 \end{vmatrix} + k\begin{vmatrix} -1 & -1 \\ 2 & 1 \end{vmatrix}$$
$$= i(0+1) - j(0+2) + k(-1+2)$$
$$= i(1) - j(2) + k(1) = \boxed{i - 2j + k}$$

SOLUTION OF EXERCISE # 6.2

Q.2: **Show that the vectors $3i - j + 7k$ and $-6i + 3j + 3k$ are at right angle to each other.**

Sol. Let $\vec{a} = (3i - j + 7k)$ & $\vec{b} = (-6i + 3j + 3k)$

$$\vec{a}\cdot\vec{b} = (3i - j + 7k).(-6i + 3j + 3k)$$

$$= (3)(-6) + (-1)(3) + (7)(3) = -18 - 3 + 21 = \boxed{0}$$

Hence $\vec{a}$ & $\vec{b}$ are right angle to each other.

Q.3: **Find the cosine of the angle between the vectors:**

(i) $\vec{a} = 2i - 8j + 3k$, $\vec{b} = 4j + 3k$ **(IIA-2017), (IA-2019)**

Sol.

$\vec{a} = 2i - 8j + 3k$

$|\vec{a}| = \sqrt{(2)^2 + (-8)^2 + (3)^2}$

$|\vec{a}| = \sqrt{4 + 64 + 9}$

$|\vec{a}| = \sqrt{77}$

$\vec{b} = 4j + 3k$

$|\vec{b}| = \sqrt{(0)^2 + (4)^2 + (3)^2}$

$|\vec{b}| = \sqrt{0 + 16 + 9}$

$|\vec{b}| = \sqrt{25} = 5$

$$\vec{a}.\vec{b} = (2i - 8j + 3k).(4j + 3k)$$

$$\vec{a}.\vec{b} = (2)(0) + (-8)(4) + (3)(3) = 0 - 32 + 9 = -23$$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|} = \frac{-23}{\sqrt{77}\,(5)} = \boxed{\frac{-23}{5\sqrt{77}}}$$

(ii) $\vec{a} = i + 2j - k$, $\vec{b} = -j - 2k$

Sol.

$\vec{a} = i + 2j - k$

$|\vec{a}| = \sqrt{(1)^2 + (2)^2 + (-1)^2}$

$|\vec{a}| = \sqrt{1 + 4 + 1} = \sqrt{6}$

$\vec{b} = -j - 2k$

$|\vec{b}| = \sqrt{(0)^2 + (-1)^2 + (-2)^2}$

$|\vec{b}| = \sqrt{0 + 1 + 4} = \sqrt{5}$

$$\vec{a}.\vec{b} = (i + 2j - k).(-j - 2k)$$

$$= (1)(0) + (2)(-1) + (-1)(-2) = 0 - 2 + 2 = 0$$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{0}{\sqrt{6}\sqrt{5}} = \boxed{0}$$

SOLUTION OF EXERCISE # 6.2

(iii) $\vec{a} = 4i + 2j - k$ & $\vec{b} = 2i + 4j - k$ (IIA-2019)

Sol.

$\vec{a} = 4i + 2j - k$

$|\vec{a}| = \sqrt{(4)^2 + (2)^2 + (-1)^2}$

$|\vec{a}| = \sqrt{16 + 4 + 1} = \sqrt{21}$

$\vec{b} = 2i + 4j - k$

$|\vec{b}| = \sqrt{(2)^2 + (4)^2 + (-1)^2}$

$|\vec{b}| = \sqrt{4 + 16 + 1} = \sqrt{21}$

$\vec{a}.\vec{b} = (4i + 2j - k).(2i + 4j - k)$

$= (4)(2) + (2)(4) + (-1)(-1) = 8 + 8 + 1 = 17$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{17}{\sqrt{21}, \sqrt{21}} = \boxed{\frac{17}{21}}$$

Q.4: If $\vec{a} = 3i + j - k$, $\vec{b} = 2i - j + k$ and $\vec{c} = 5i + 3k$, find $(2\vec{a} + \vec{b}).\vec{c}$

Sol. $2\vec{a} + \vec{b} = 2(3i + j - k) + (2i - j + k)$

$= 6i + 2j - 2k + 2i - j + k = 8i + j - k$

$(2\vec{a} + \vec{b}).\vec{c} = (8i + j - k).(5i + 3k)$

$= (8)(5) + (1)(0) + (-1)(3) = 40 + 0 - 3 = \boxed{37}$

Q.5: What is the cosine of the angle between $\overrightarrow{P_1P_2}$ and $\overrightarrow{P_3P_4}$ if $P_1(2, 1, 3)$, $P_2(-4, 4, 5)$, $P_3(0, 7, 0)$ & $P_4(-3, 4, -2)$.

Sol.

$\overrightarrow{P_1P_2} = [-4, 4, 5] - [2, 1, 3]$

$\overrightarrow{P_1P_2} = [-6, 3, 2]$

$\overrightarrow{P_1P_2} = -6i + 3j + 2k$

$|\overrightarrow{P_1P_2}| = \sqrt{(-6)^2 + (3)^2 + (2)^2}$

$|\overrightarrow{P_1P_2}| = \sqrt{36 + 9 + 4}$

$|\overrightarrow{P_1P_2}| = \sqrt{49} = 7$

$\overrightarrow{P_3P_4} = [-3, 4, -2] - [0, 7, 0]$

$\overrightarrow{P_3P_4} = [-3, -3, -2]$

$\overrightarrow{P_3P_4} = -3i - 3j - 2k$

$|\overrightarrow{P_3P_4}| = \sqrt{(-3)^2 + (-3)^2 + (-2)^2}$

$|\overrightarrow{P_3P_4}| = \sqrt{9 + 9 + 4}$

$|\overrightarrow{P_3P_4}| = \sqrt{22}$

SOLUTION OF EXERCISE # 6.2

$\overline{P_1P_2}.\overline{P_3P_4} = (-6i + 3j + 2k).(-3i - 3j - 2k)$

$= (-6)(-3) + (3)(-3) + (2)(-2) = 18 - 9 - 4 = 5$

$$\cos\theta = \frac{\overline{P_1P_2}.\overline{P_3P_4}}{|\overline{P_1P_2}||\overline{P_3P_4}|} = \boxed{\frac{5}{7\sqrt{22}}}$$

Q.6: If $\vec{a} = [a_1, a_2, a_3]$ and $\vec{b} = [b_1, b_2, b_3]$, Prove that $\vec{a}.\vec{b} = \frac{1}{2}\left[|\vec{a}+\vec{b}|^2 - |\vec{a}|^2 - |\vec{b}|^2\right]$

Sol. $\text{R.H.S} = \frac{1}{2}\left[|\vec{a}+\vec{b}|^2 - |\vec{a}|^2 - |\vec{b}|^2\right]$

$$= \frac{1}{2}\left[(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) - \vec{a}.\vec{a} - \vec{b}.\vec{b}\right]$$

$$= \frac{1}{2}\left[\cancel{\vec{a}.\vec{a}} + \vec{a}.\vec{b} + \vec{b}.\vec{a} + \cancel{\vec{b}.\vec{b}} - \cancel{\vec{a}.\vec{a}} - \cancel{\vec{b}.\vec{b}}\right]$$

$$= \frac{1}{2}\left[\vec{a}.\vec{b} + \vec{b}.\vec{a}\right]$$

$$= \frac{1}{2}\left[\vec{a}.\vec{b} + \vec{a}.\vec{b}\right] \qquad \because \vec{a}.\vec{b} = \vec{b}.\vec{a}$$

$$= \frac{1}{2}\left[2.(\vec{a}.\vec{b})\right] = \vec{a}.\vec{b} = \text{R.H.S.}$$ **Proved**

Q.7: Find $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})$ if $\vec{a} = i + 2j + 3k$ and $\vec{b} = 2i - j + k$

Sol. $\vec{a}+\vec{b} = (i + 2j + 3k) + (2i - j + k)$

$= i + 2j + 3k + 2i - j + k = 3i + j + 4k$

$\vec{a}-\vec{b} = (i + 2j + 3k) - (2i - j + k)$

$= i + 2j + 3k - 2i + j - k = -i + 3j + 2k$

$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = (3i + j + 4k).(-i + 3j + 2k)$

$= (3)(-1) + (1)(3) + (4)(2) = -\cancel{3} + \cancel{3} + 8 = \boxed{8}$

SOLUTION OF EXERCISE # 6.2

Q.8: Prove that for every pair of vectors $\vec{a}$ and $\vec{b}$

$$\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=|\vec{a}|^2-|\vec{b}|^2$$

Sol. L.H.S. $=\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)$

$=\vec{a}.\vec{a}-\vec{a}.\vec{b}+\vec{b}.\vec{a}-\vec{b}.\vec{b}$

$=\vec{a}.\vec{a}-\cancel{\vec{a}.\vec{b}}+\cancel{\vec{a}.\vec{b}}-\vec{b}.\vec{b}$ $\quad\because \vec{a}.\vec{b}=\vec{b}.\vec{a}$

$=\vec{a}.\vec{a}-\vec{b}.\vec{b}$

$=|\vec{a}|^2-|\vec{b}|^2=$ R.H.S. **Proved**

Q.9: Find x so that $\vec{a}$ and $\vec{b}$ are perpendicular.

(i) $\vec{a}=2i+4j-7k,\quad \vec{b}=2i+6j+xk$

(IIA-2019), (IIA-2021)

Sol. As $\vec{a}$ & $\vec{b}$ are perpendicular, so

$\vec{a}.\vec{b}=0$

$(2i+4j-7k).(2i+6j+xk)=0$

$(2)(2)+(4)(6)+(-7)(x)=0$

$4+24-7x=0$

$28-7x=0$

$-7x=-28$

$x=\frac{-28}{-7} \quad\Rightarrow\quad \boxed{x=4}$

(ii) $\vec{a}=xi-2j+5k,\ \vec{b}=2i-j+3k$

Sol. As $\vec{a}$ & $\vec{b}$ are perpendicular, so

$\vec{a}.\vec{b}=0$

$(xi-2j+5k).(2i-j+3k)=0$

$2x+2+15=0$

$2x+17=0$

$2x=-17 \quad\Rightarrow\quad \boxed{x=\frac{-17}{2}}$

SOLUTION OF EXERCISE # 6.2

Q.10: If $\vec{a} = 2i - 3j + 4k$ and $\vec{b} = 2j + 4k$. Find the component or projection of $\vec{a}$ along $\vec{b}$. (IIA-2016)

Sol.

$\vec{a}.\vec{b}$

$= (2i - 3j + 4k) . (2j + 4k$

$= (2)(0) + (-3)(2) + (4)(4)$

$= (0 - 6 + 16)$

$= 10$

$|\vec{b}| = \sqrt{(0)^2 + (2)^2 + (4)^2}$

$= \sqrt{0 + 4 + 16}$

$= \sqrt{20} = \sqrt{4 \times 5}$

$= 2\sqrt{5}$

$$\text{Projection of } \vec{a} \text{ along } \vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{10}{2\sqrt{5}} = \frac{5}{\sqrt{5}} = \boxed{\sqrt{5}}$$

Q.11: Under what condition does the relation $(\vec{a}.\vec{b})^2 = |\vec{a}|^2\ |\vec{b}|^2$ hold for two vectors $\vec{a}$ and $\vec{b}$.

Sol. As, $(\vec{a}.\vec{b})^2 = |\vec{a}|^2\ |\vec{b}|^2$

$$(|\vec{a}||\vec{b}|\cos\theta)^2 = |\vec{a}|^2\ |\vec{b}|^2 \quad \because \left\{\begin{matrix}\text{By Definition} \\ \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta\end{matrix}\right\}$$

$$|\vec{a}|^2\ |\vec{b}|^2 \cos^2\theta = |\vec{a}|^2\ |\vec{b}|^2$$

$$\cos^2\theta = \frac{|\vec{a}|^2\ |\vec{b}|^2}{|\vec{a}|^2\ |\vec{b}|^2}$$

$$\cos^2\theta = 1$$

$$\sqrt{\cos^2\theta} = \pm\sqrt{1} \quad \Rightarrow \quad \cos\theta = \pm 1$$

Either $\cos\theta = 1$ OR $\cos\theta = -1$

$\theta = \cos^{-1}(1)$

$\theta = 0^\circ$

$\theta = 0 \times \frac{\pi}{180}$

$\theta = 0$ rad

$\theta = \cos^{-1}(-1)$

$\theta = 180^\circ$

$\theta = 180 \times \frac{\pi}{180}$

$\theta = \pi$ rad

$$\{(0, \pi)\}$$

SOLUTION OF EXERCISE # 6.2

Q.12: If the vectors $3i + j - k$ and $\lambda i - 4j + 4k$ are parallel, find value of λ.

(IIA-2019), (IIA-2020), (IA-2021), (IIA-2021)

Sol. As $\vec{a}$ and $\vec{b}$ are parallel, so $\vec{a}\times\vec{b}=0$

$$\Rightarrow \begin{vmatrix} i & j & k \\ 3 & 1 & -1 \\ \lambda & -4 & 4 \end{vmatrix} = 0$$

$$\Rightarrow i\begin{vmatrix} 1 & -1 \\ -4 & 4 \end{vmatrix} - j\begin{vmatrix} 3 & -1 \\ \lambda & 4 \end{vmatrix} + k\begin{vmatrix} 3 & 1 \\ \lambda & -4 \end{vmatrix} = 0$$

$$\Rightarrow i(4-4) - j(12+\lambda) + k(-12-\lambda) = 0$$

$$\Rightarrow i(0) - (12+\lambda)j + (-12-\lambda)k = 0i + 0j + 0k$$

Comparing Coefficient of j, we get

$$-(12+\lambda) = 0$$

$$-12-\lambda = 0$$

$$-\lambda = 12 \Rightarrow \boxed{\lambda = -12}$$

Q.13: If $\vec{a} = i - 2j + k$, $\vec{b} = i + 2j - 4k$ & $\vec{c} = 2i - 3j + k$

Evaluate. (i) $(\vec{a}\times\vec{b}).(\vec{a}\times\vec{c})$

Sol.

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 1 & 2 & -4 \end{vmatrix}$$

$$= i\begin{vmatrix} -2 & 1 \\ 2 & -4 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ 1 & -4 \end{vmatrix} + k\begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix}$$

$$= i(8-2) - j(-4-1) + k(2+2)$$

$$= 6i + 5j + 4k$$

$$\vec{a}\times\vec{c} = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= i\begin{vmatrix} -2 & 1 \\ -3 & 1 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + k\begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix}$$

$$= i(-2+3) - j(1-2) + k(-3+4)$$

$$= i + j + k$$

$$(\vec{a}\times\vec{b}).(\vec{a}\times\vec{c}) = (6i+5j+4k).(i+j+k)$$

$$= (6)(1) + (5)(1) + (4)(1) = 6+5+4 = \boxed{15}$$

SOLUTION OF EXERCISE # 6.2

(ii) $(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c})$

Sol. $(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c}) = \begin{vmatrix} i & j & k \\ 6 & 5 & 4 \\ 1 & 1 & 1 \end{vmatrix}$

$$= i\begin{vmatrix} 5 & 4 \\ 1 & 1 \end{vmatrix} - j\begin{vmatrix} 6 & 4 \\ 1 & 1 \end{vmatrix} + k\begin{vmatrix} 6 & 5 \\ 1 & 1 \end{vmatrix}$$

$$= i(5-4) - j(6-4) + k(6-5) = \boxed{i-2j+k}$$

Q.14: If $\vec{a} = i+3j-7k$ and $\vec{b} = 5i-2j+4k$, find:

(i) $\vec{a}.\vec{b}$

Sol. $\vec{a}.\vec{b} = (i+3j-7k).(5i-2j+4k)$

$$= (1)(5)+(3)(-2)+(-7)(4) = 5-6-28 = \boxed{-29}$$

(ii) $\vec{a}\times\vec{b}$

Sol. $\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 1 & 3 & -7 \\ 5 & -2 & 4 \end{vmatrix}$

$$= i\begin{vmatrix} 3 & -7 \\ -2 & 4 \end{vmatrix} - j\begin{vmatrix} 1 & -7 \\ 5 & 4 \end{vmatrix} + k\begin{vmatrix} 1 & 3 \\ 5 & -2 \end{vmatrix}$$

$$= i(12-14) - j(4+35) + k(-2-15)$$

$$= i(-2) - j(39) + k(-17) = \boxed{-2i-39j-17k}$$

(iii) Direction cosines of $\vec{a}\times\vec{b}$

Sol. $|\vec{a}\times\vec{b}| = \sqrt{(-2)^2+(-39)^2+(-17)^2}$

$$= \sqrt{4+1521+289} = \sqrt{1814}$$

Direction cosines of $(\vec{a}\times\vec{b})$ are: $\boxed{\dfrac{-2}{\sqrt{1814}}, \dfrac{-39}{\sqrt{1814}}, \dfrac{-17}{\sqrt{1814}}}$

SOLUTION OF EXERCISE # 6.2

Q.15: Prove that for the vector $\vec{a}$ & $\vec{b}$

(i) $|\vec{a}\times\vec{b}|^2 + |\vec{a}.\vec{b}|^2 = |\vec{a}|^2\,|\vec{b}|^2$ (IIA-2020)

Sol. L.H.S. $= \left|\vec{a}\times\vec{b}\right|^2 + \left|\vec{a}.\vec{b}\right|^2$

$= \left[|\vec{a}||\vec{b}|\sin\theta\right]^2 + \left[|\vec{a}||\vec{b}|\cos\theta\right]^2$ {By Definition}

$= |\vec{a}|^2\,|\vec{b}|^2\sin^2\theta + |\vec{a}|^2\,|\vec{b}|^2\cos^2\theta$

$= |\vec{a}|^2\,|\vec{b}|^2\left(\sin^2\theta+\cos^2\theta\right)$

$= |\vec{a}|^2\,|\vec{b}|^2(1)$ $\because\{\sin^2\theta+\cos^2\theta=1\}$

$= |\vec{a}|^2\,|\vec{b}|^2 =$ R.H.S. **Proved.**

(ii) $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)$

Sol. L.H.S. $=\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)$

$= \vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}$

$= 0+\vec{a}\times\vec{b}+\vec{a}\times\vec{b}-0$ $\because\left\{\begin{array}{l}\vec{a}\times\vec{a}=\vec{b}\times\vec{b}=0\\ \&\ \vec{a}\times\vec{b}=-\vec{b}\times\vec{a}\end{array}\right\}$

$= 2(\vec{a}\times\vec{b}) =$ R.H.S. **Proved.**

Q.16: Prove that for vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.

$[\vec{a}\times(\vec{b}+\vec{c})]+[\vec{b}\times(\vec{c}+\vec{a})]+[\vec{c}\times(\vec{a}+\vec{b})]=0$

Sol. L.H.S. $= [\vec{a}\times(\vec{b}+\vec{c})]+[\vec{b}\times(\vec{c}+\vec{a})]+[\vec{c}\times(\vec{a}+\vec{b})]$

$= \vec{a}\times\vec{b}+\vec{a}\times\vec{c}+\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}+\vec{c}\times\vec{b}$

$= \cancel{\vec{a}\times\vec{b}}+\cancel{\vec{a}\times\vec{c}}+\cancel{\vec{b}\times\vec{c}}-\cancel{\vec{a}\times\vec{b}}-\cancel{\vec{a}\times\vec{c}}-\cancel{\vec{b}\times\vec{c}}$

$= 0 =$ R.H.S. **Proved.**

Q.17: Find a vector perpendicular to both the lines $\overrightarrow{AB}$ and $\overrightarrow{CD}$, where A is (0, 2, 4), B is (3, -1, 2), C is (2, 0, 1) and D is (4, 2, 0).

Sol. $\overrightarrow{AB}=[3,-1,2]-[0,2,4]=[3,-3,-2]=3\hat{i}-3\hat{j}-2\hat{k}$

$\overrightarrow{CD}=[4,2,0]-[2,0,1]=[2,2,-1]=2\hat{i}+2\hat{j}-\hat{k}$

SOLUTION OF EXERCISE # 6.2

Vector perpendicular to both AB & CD is

$$\overrightarrow{AB}\times\overrightarrow{CD}=\begin{vmatrix} i & j & k \\ 3 & -3 & -2 \\ 2 & 2 & -1 \end{vmatrix} = i\begin{vmatrix} -3 & -2 \\ 2 & -1 \end{vmatrix} - j\begin{vmatrix} 3 & -2 \\ 2 & -1 \end{vmatrix} + k\begin{vmatrix} 3 & -3 \\ 2 & 2 \end{vmatrix}$$

$$= i(3+4) - j(-3+4) + k(6+6) = \boxed{7i - j + 12k}$$

Q.18: Find $\left|\left(\vec{a}\times\vec{b}\right)\times c\right|$ (IA-2018)

if $\vec{a} = i - 2j - 3k,\ \vec{b} = 2i + j - k,\ \vec{c} = i + 3j - 2k.$

Sol.

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 1 & -2 & -3 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= i\begin{vmatrix} -2 & -3 \\ 1 & -1 \end{vmatrix} - j\begin{vmatrix} 1 & -3 \\ 2 & -1 \end{vmatrix} + k\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}$$

$$= i(2+3) - j(-1+6) + k(1+4)$$

$$= 5j - 5j + 5k$$

$$\left(\vec{a}\times\vec{b}\right)\times\vec{c} = \begin{vmatrix} i & j & k \\ 5 & -5 & 5 \\ 1 & 3 & -2 \end{vmatrix}$$

$$= i\begin{vmatrix} -5 & 5 \\ 3 & -2 \end{vmatrix} - j\begin{vmatrix} 5 & 5 \\ 1 & -2 \end{vmatrix} + k\begin{vmatrix} 5 & -5 \\ 1 & 3 \end{vmatrix}$$

$$= i(10-15) - j(-10-5) + k(15+5)$$

$$= -5i + 15j + 20k$$

$$\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right| = \sqrt{(5)^2 + (15)^2 + (20)^2}$$

$$= \sqrt{25+225+400} = \sqrt{650} = \sqrt{25\times 26} = \boxed{5\sqrt{26}}$$

Q.19: Find the sine of the angle and the unit vector perpendicular to each:

(i) $\vec{a} = i + j + k$ and $\vec{b} = 2i + 3j - k$

Sol.

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2}$$

$$|\vec{a}| = \sqrt{1+1+1}$$

$$|\vec{a}| = \sqrt{3}$$

$$|\vec{b}| = \sqrt{(2)^2 + (3)^2 + (-1)^2}$$

$$|\vec{b}| = \sqrt{4+9+1}$$

$$|\vec{b}| = \sqrt{14}$$

SOLUTION OF EXERCISE # 6.2

$$\vec{a}\times\vec{b}=\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & 3 & -1 \end{vmatrix}=i\begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix}-j\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}+k\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$

$$=i(-1-3)-j(-1-2)+k(3-2)$$

$$=i(-4)-j(-3)+k(1)=-4i+3j+k$$

$$\left|\vec{a}\times\vec{b}\right|=\sqrt{(-4)^2+(3)^2+(1)^2}=\sqrt{16+9+1}=\sqrt{26}$$

$$\sin\theta=\frac{\left|\vec{a}\times\vec{b}\right|}{|\vec{a}|\ |\vec{b}|}=\frac{\sqrt{26}}{\sqrt{3}.\sqrt{14}}=\frac{\sqrt{26}}{\sqrt{42}}=\sqrt{\frac{26}{42}}=\boxed{\sqrt{\frac{13}{21}}}$$

Units vector perpendicular to both $\vec{a}$ & $\vec{b}$ is :

$$\hat{u}=\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}=\boxed{\frac{-4i+3j+k}{\sqrt{26}}}$$

(ii) $\vec{a}=2i-j+k$ and $\vec{b}=3i+4j-k$ (IA-2017), (IA-2021)

Sol.

$$|\vec{a}|=\sqrt{(2)^2+(-1)^2+(1)^2}$$
$$|\vec{a}|=\sqrt{4+1+1}$$
$$|\vec{a}|=\sqrt{6}$$

$$|\vec{b}|=\sqrt{(3)^2+(4)^2+(-1)^2}$$
$$|\vec{b}|=\sqrt{9+16+1}$$
$$|\vec{b}|=\sqrt{26}$$

$$\vec{a}\times\vec{b}=\begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix}=i\begin{vmatrix} -1 & 1 \\ 4 & -1 \end{vmatrix}-j\begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix}+k\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$$

$$=i(1-4)-j(-2-3)+k(8+3)=-3i+5j+11k$$

$$|\vec{a}\times\vec{b}|=\sqrt{(-3)^2+(5)^2+(11)^2}=\sqrt{9+25+121}=\sqrt{155}$$

$$\sin\theta=\frac{\vec{a}\times\vec{b}}{|\vec{a}|\ |\vec{b}|}=\frac{\sqrt{155}}{\sqrt{6}\sqrt{26}}=\boxed{\frac{\sqrt{155}}{\sqrt{156}}}$$

SOLUTION OF EXERCISE # 6.2

Units vector perpendicular to both $\vec{a}$ & $\vec{b}$ is :

$$\hat{u} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \boxed{\frac{-3i+5j+11k}{\sqrt{155}}}$$

Q.20: Given $\vec{a} = 2i - j$ and $\vec{b} = j + k$, if $|\vec{c}| = 12$ and $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, write the component form of $\vec{c}$.

Sol. $\vec{a} = 2i - j,\quad \vec{b} = j + k,\quad |\vec{c}| = 12$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= i\begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix}$$

$$= i(-1-0) - j(2-0) + k(2+0) = -i - 2j + 2k$$

$$|\vec{a}\times\vec{b}| = \sqrt{(-1)^2 + (-2)^2 + (2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$$

As, 'c' is perpendicular to both $\vec{a}$ & $\vec{b}$ is :

$$\text{So,}\quad \frac{\vec{c}}{|\vec{c}|} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$

$$\frac{\vec{c}}{12} = \frac{-i-2j+2k}{3}$$

$$\vec{c} = \frac{12(-i-2j+2k)}{3} = 4(-i-2j+2k) = \boxed{-4i-8j+8k}$$

Q.21: Using cross product, find the area of each triangle whose vertices have the following co-ordinates:

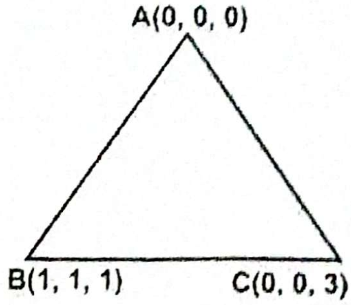
(i) $(0, 0, 0),\quad (1, 1, 1),\quad (0, 0, 3)$ **(IIA-2018)**

Sol. Let $A(0, 0, 0),\ B(1, 1, 1),\ C(0, 0, 3)$

$$\vec{AB} = [1, 1, 1] - [0, 0, 0] = [1, 1, 1]$$

SOLUTION OF EXERCISE # 6.2

$\overrightarrow{AC} = [0, 0, 3] - [0, 0, 0] = [0, 0, 3]$



$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 0 & 0 & 3 \end{vmatrix}$$

$$= i\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} + k\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}.$$

$$= i(3-0) - j(3-0) + k(0,-0) = 3i - 3j + 0k$$

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(3)^2 + (-3)^2 + (0)^2} = \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$$

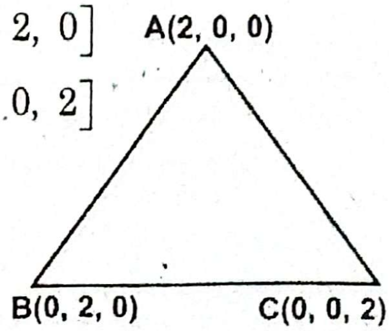
$$\text{Area of Triangle} = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \boxed{\frac{3\sqrt{2}}{2}\ \text{sq. unit}}$$

(ii) $(2, 0, 0)$, $(0, 2, 0)$, $(0, 0, 2)$ **(IA-2022)**

Sol. Let $A(2, 0, 0)$, $B(0, 2, 0)$, $C(0, 0, 2)$

$\overrightarrow{AB} = [0, 2, 0] - [2, 0, 0] = [-2, 2, 0]$

$\overrightarrow{AC} = [0, 0, 2] - [2, 0, 0] = [-2, 0, 2]$



$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{vmatrix}$$

$$= i\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} - j\begin{vmatrix} -2 & 0 \\ -2 & 2 \end{vmatrix} + k\begin{vmatrix} -2 & 2 \\ -2 & 0 \end{vmatrix}$$

$$= i(4-0) - j(-4+0) + k(0+4) = 4i + 4j + 4k$$

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(4)^2 + (4)^2 + (4)} = \sqrt{16+16+16}$$

$$= \sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$$

$$\text{Area of triangle} = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}(4\sqrt{3}) = \boxed{2\sqrt{3}\ \text{sq.unit}}$$

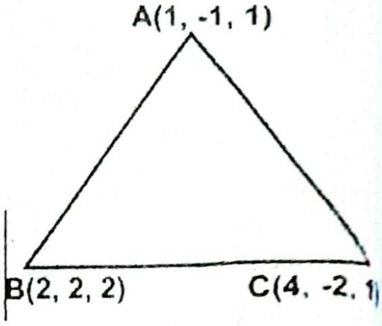
(iii) $(1, -1, 1)$, $(2, 2, 2)$, $(4, -2, 1)$

SOLUTION OF EXERCISE # 6.2

Sol. Let $A(1,-1,1)$, $B(2,2,2)$, $C(4,-2,1)$

$\overrightarrow{AB} = [2,2,2]-[1,-1,1] = [1,3,1]$

$\overrightarrow{AC} = [4,-2,1]-[1,-1,1] = [3,-1,0]$



$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 1 & 3 & 1 \\ 3 & -1 & 0 \end{vmatrix}$$

$$= i\begin{vmatrix} 3 & 1 \\ -1 & 0 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ 3 & 0 \end{vmatrix} + k\begin{vmatrix} 1 & 3 \\ 3 & -1 \end{vmatrix}$$

$$= i(0+1)-j(0-3)+k(-1-9)$$

$$= i+3j-10k$$

$$|\overrightarrow{AB}\times\overrightarrow{AC}| = \sqrt{(1)^2+(3)^2+(-10)^2} = \sqrt{1+9+100} = \sqrt{110}$$

$$\text{Area of triangle} = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}| = \boxed{\frac{\sqrt{110}}{2}\ \text{sq.unit}}$$

Q.22: Find the area of parallelogram determined by the vectors: $\vec{a} = i+2j+3k$ and $\vec{b} = -3i-2j+k$.

Sol.

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{vmatrix}$$

$$= i\begin{vmatrix} 2 & 3 \\ -2 & 1 \end{vmatrix} - j\begin{vmatrix} 1 & 3 \\ -3 & 1 \end{vmatrix} + k\begin{vmatrix} 1 & 2 \\ -3 & -2 \end{vmatrix}$$

$$= i(2+6)-j(1+9)+k(-2+6)$$

$$= 8i-10j+4k$$

$$|\vec{a}\times\vec{b}| = \sqrt{(8)^2+(-10)^2+(4)^2} = \sqrt{64+100+16} = \sqrt{180}$$

$$\text{Area of parallelogram} = |\vec{a}\times\vec{b}| = \boxed{\sqrt{180}\ \text{sq.unit}}$$